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SIMULATION OF TRANSIENT GAS FLOWS IN NETWORKS

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SUMMARY

A technique is presented for calculating the transient flow in high pressure transportation systems where both simple systems (without compressors) and systems with compressors have been taken into consideration. A partial differential equation characterizing the dynamic gas flow through a pipeline and a numerical scheme for its solution are considered. A method of computing node pressures is also characterized.

KEY WORDS Gas Networks Computational Methods Simulation Optimization

INTRODUCTION

A gas network is in the transient state when the values of quantities characterizing the supply of gas to the system or its consumption are functions of time. Simulation of transient flow in gas networks is necessary both for design and for control. Dynamic simulation of a gas network requires a suitable mathematical model and a numerical method for its solution. An explicit model of a dynamic, physical 'real-world' system, such as gas flowing through a pipeline, is a set of partial differential equations written on the basis of:

(a) the principle of conservation of mass
(b) the equation of state
(c) the equation of conservation of momentum.

By assuming given intervals for quantitative and qualitative changes of particular quantities, the final equations which define the phenomenon exactly can be approximated by simpler equations.

Since a sophisticated model causes difficulties in simulating a pipeline system, it must be stripped of its complexities. The superficialities should be deleted, while maintaining the model concepts, which are pragmatically and operationally defined. The numerical solution of the partial differential equations which characterize a dynamic model of a network takes much computation time. The problem is to find, for a given mathematical model of a pipeline, a numerical method which meets the criteria of accuracy and relatively small computation time.

MODEL ANALYSIS

In the development of the explicit mathematical model for the dynamics of gas flowing through a pipeline, it was assumed that:

(i) the flow is turbulent
(ii) the gas process is isothermal

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(iii) the pipeline is rectilinear (i.e. the curvature radii are great)
(iv) the pipeline cross-section area is constant.

These assumptions are used by Czernyj[1] in the explicit description of the gas dynamics in a pipeline by means of the following set of non-linear partial differential equations.

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} &= \frac{\partial(\rho w)}{\partial t} + \frac{\lambda}{2D}\rho w^2 + \frac{\partial}{\partial x}((1+\beta)\rho w^2) + g\rho \sin\alpha \\ -\frac{\partial p}{\partial t} &= c^2 \frac{\partial(\rho w)}{\partial x} \end{aligned}\right\} \tag{1}$$

where

c is the speed of sound in gas, m/s
$w = w(x, t)$ is the average gas velocity (averaged over cross-section area) in pipeline, m/s
$\rho = \rho(x, t)$ is the average gas density (averaged over cross-section area) in pipeline, kg/m^3
$g = 9{\cdot}81$ is the acceleration due to gravity, m/s^2
$\alpha(^\circ)$ is the angle of pipeline inclination with respect to a horizontal plane
λ is the friction coefficient for fluid in the pipeline
D is the pipeline diameter, m
$p = p(x, t)$ is the average gas pressure (averaged over cross-section area) in pipeline, Pa
β is the correction of Coriolis to allow for a non-uniform velocity profile in the stream.

The constituent factors $\frac{\partial(\rho w)}{\partial t}$, $\frac{\lambda \rho w^2}{2D}$ and $\rho g \sin\alpha$ define the gas inertia and friction force of gravity, respectively. The factor $(1+\beta)\rho w^2$ is determined by the flowing-gas dynamic pressure. In practice, it is assumed that the pipelines are run horizontally; thus $\rho g \sin\alpha = 0$ and because $\beta = 0$ for the turbulent flows, the set of equations (1) may be rewritten as

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} &= \frac{\partial(\rho w)}{\partial t} + \frac{\lambda}{2D}\rho w^2 + \frac{\partial(\rho w^2)}{\partial x} \\ -\frac{\partial p}{\partial t} &= c^2 \frac{\partial(\rho w)}{\partial x} \end{aligned}\right\} \tag{2}$$

By integrating the first of equations (2) between $x = 0$ and $x = L$ (where L is the length of the pipe) and multiplying by dx we get

$$p(0, t) - p(L, t) = \int_0^L \frac{\partial(\rho w)}{\partial t}\,dx + \int_0^L \frac{\lambda}{2}\frac{w^2}{D}\rho\,dx + (\rho w^2)_{x=L} - (\rho w^2)_{x=0} \tag{3}$$

By computing the values

$$\delta_1 = \frac{\int_0^L \frac{\partial(\rho w)}{\partial t}\,dx}{p(0, t) - p(L, t)} \times 100 \text{ per cent}$$

$$\delta_2 = \frac{\int_0^L \frac{\lambda}{2}\frac{w^2}{D}\rho\,dx}{p(0, t) - p(L, t)} \times 100 \text{ per cent}$$

$$\delta_3 = \frac{(\rho w^2)_{x=L} - (\rho w^2)_{x=0}}{p(0, t) - p(L, t)} \times 100 \text{ per cent}$$

we can confirm whether it is possible to reduce the set of equations (2) to a simpler form.

The term $\int_0^L \frac{\partial(\rho w)}{\partial t} dx$ may be transformed to the following form:

$$\frac{1}{S}\left(\frac{\partial(\rho w S)}{\partial t}\right)_{\text{average}} L \quad \text{where} \quad S = \frac{\pi D^2}{4}$$

Next

$$\left(\frac{\partial(\rho w S)}{\partial t}\right)_{\text{average}} \simeq \frac{\Delta Q_m}{\Delta t} = \frac{\rho^* \Delta Q_v^*}{\Delta t}$$

where an asterisk indicates values under NTP conditions, i.e. 273°K and 0·1 MPa, and

Q_m is the mass flow (kg/s)
Q_v is the volume flow (m^3/s)
Δt is the discretization time for the function $Q_v^*(L, t)$ (Figure 1) (in the analysed case $\Delta t = 2$ h)

It was assumed that $\Delta G_v^* = \Delta Q_{v\,max}^*$. Using a boundary condition (Figure 1) we get:

$$\Delta Q_{v\,max}^* = 0{\cdot}15 Q_{v\,max}^*$$

The values of the gas velocity and gas density are calculated respectively from the formulae

$$w = \frac{Q_v^* p^* T}{S p T^*}$$

$$\rho = \rho^* \frac{p}{p^*}$$

(i) For $L = 10^4$ m, $p_0 = 4$ MPa, $D = 0{\cdot}7$ m, $p_L = 3{\cdot}8$ MPa, (where p_0 is the pressure at $x = 0$ and p_L is the pressure at $x = L$) and $Q_v^* = 14{\cdot}22$ m^3/s

$$\delta_1 = 0{\cdot}587 \text{ per cent}$$
$$\delta_2 = 171{\cdot}2 \text{ per cent}$$
$$\delta_3 = 0{\cdot}036 \text{ per cent}$$

(ii) For $L = 5 \times 10^4$ m, $p_0 = 5$ MPa, $D = 0{\cdot}7$ m, $p_L = 4{\cdot}7$ MPa and $Q_v^* = 90{\cdot}28$ m^3/s

$$\delta_1 = 0{\cdot}513 \text{ per cent}$$
$$\delta_2 = 162{\cdot}3 \text{ per cent}$$
$$\delta_3 = 0{\cdot}021 \text{ per cent}$$

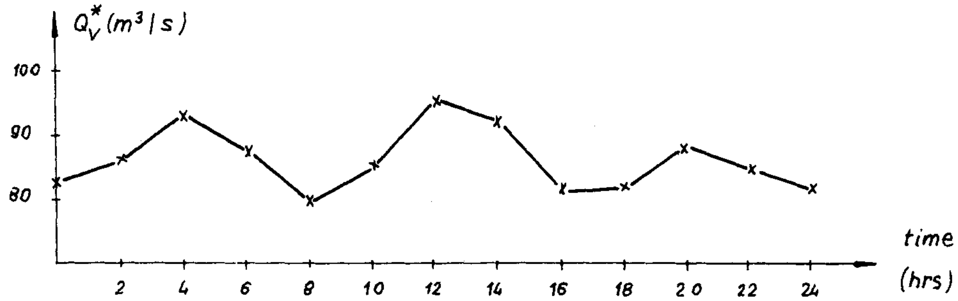

Figure 1. Changes of flow with time (boundary condition)

(iii) For $L = 10^5$ m, $p_0 = 5$ MPa, $D = 0{\cdot}7$ m, $p_L = 4{\cdot}3$ MPa and $Q_v^* = 94{\cdot}44\ \mathrm{m^3/s}$

$$\delta_1 = 0{\cdot}380 \text{ per cent}$$
$$\delta_2 = 151{\cdot}2 \text{ per cent}$$
$$\delta_3 = 0{\cdot}019 \text{ per cent}$$

By comparing the values of δ_1, δ_2 and δ_3 we see that we can neglect terms

$$\frac{\partial(\rho w^2)}{\partial x} \quad \text{and} \quad \frac{\partial(\rho w)}{\partial t}$$

Finally we obtain

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} &= \frac{\lambda}{2}\frac{w^2}{D}\rho \\ -\frac{\partial p}{\partial t} &= c^2\frac{\partial(\rho w)}{\partial x} \end{aligned}\right\} \tag{4}$$

Using the relations

$$p = c^2\rho$$
$$\rho Q_v = Q_m = \rho^* Q_v^*$$

we transform the set of equations (4) into

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= -\frac{\lambda\rho^{*2}c^2}{2DS^2}\frac{Q_v^{*2}}{p} \\ \frac{\partial Q_v^*}{\partial x} &= -\frac{S}{\rho^* c^2}\frac{\partial p}{\partial t} \end{aligned}\right\} \tag{5}$$

By differentiating the first equation with respect to x and substituting the second equation into the first we get

$$\frac{\partial p^2}{\partial t} = a\frac{\partial^2 p^2}{\partial x^2} \tag{6}$$

where

$$a = \frac{DSc^2}{\lambda Q_v}$$

Because it was assumed that $Q_v(x, t)$ is averaged along the pipe for each interval of time Δt, equation (6) is linear with respect to the square of pressure for each interval Δt. It is possible to show in another way that the term $\partial(\rho w^2)/\partial x$ is small when compared to the other terms and may be discarded. It is possible to reformulate the first equation of set (2) in the following way:

$$-\frac{\partial(c^2\rho)}{\partial x} - \frac{\partial(w^2\rho)}{\partial x} = \frac{\partial(\rho w)}{\partial t} + \frac{\lambda\rho w^2}{2D} \tag{7}$$

Next

$$-\frac{\partial}{\partial x}[\rho(c^2 + w^2)] = \frac{\partial(\rho w)}{\partial t} + \frac{\lambda\rho w^2}{2D}$$

Thus

$$-\frac{\partial}{\partial x}\left[p\left(1 + \frac{w^2}{c^2}\right)\right] = \frac{\partial(\rho w)}{\partial t} + \frac{\lambda\rho w^2}{2D}$$

Taking into account the fact that the highest flow velocity is not higher than 20 m/s and assuming $c = 300$ m/s, we can approximate the value of the expression

$$1+\frac{w^2}{c^2}=1{\cdot}00444\cong 1$$

to unity. Thus

$$-\frac{\partial p}{\partial x}=\frac{\partial(\rho w)}{\partial t}+\frac{\lambda \rho w^2}{2D} \tag{8}$$

Next the rightness of neglecting the component $\partial(\rho w)/\partial t$ will be estimated by using a comparison of the results obtained from the following set of equations:

$$\left.\begin{aligned}\frac{\partial Q_v^*}{\partial t}&=-\frac{S}{\rho^*}\frac{\partial p}{\partial x}-\frac{\lambda c^2\rho^*}{2DS}\frac{Q^{*2}}{p}\\ \frac{\partial p}{\partial t}&=-\frac{\rho^* c^2}{s}\frac{\partial Q_v^*}{\partial x}\end{aligned}\right\} \tag{9}$$

which satisfy the set of equations (2) without term $\partial(\rho w^2)/\partial x$, with results which were obtained using equation (6) for the same boundary conditions.

It was assumed that at the initial moment $t=0$ we have steady-state flow along the pipeline; thus

$$Q_v^*(x,0)=Q_{v,0}^*=\text{const}$$

It was assumed that the pipeline under investigation was supplied from a compressor station ($x=0$) and was loaded with a receiver ($x=L$) having a load $Q_v^*(t)$ varying in time. It was assumed that by appropriately changing the capacity of the compressor station (changing the number of machines operating simultaneously and/or operating parameters of any of them), the value of the pressure at the beginning of the pipeline is kept invariant and equal to the allowable maximum. Thus

$$\left.\begin{aligned}p(0,t)&=p_0=\text{const}\\ Q_v^*(L,t)&=f(t)\end{aligned}\right\} \tag{10}$$

To determine the form of the function $Q_v^*(L,t)$ a statistical analysis was performed for the 24-hour reports covering a period of a year (the reports contained the values of pressures and flow rates for the selected points in the pipeline system). Next, a most probable flow change at the pipeline end was defined. The solution, $Q_v^*(L,t)$ is a sampling function with a sampling period of $\Delta t = 2$ h, the time interval being $t\in[0, 24]$. While using this function as a boundary condition it was assumed that the function is linear in the intervals under consideration (Figure 1).

Equations (6) and (9) were solved using the method of lines. Each of the partial differential equations was replaced by a set of ordinary differential equations. Assuming boundary conditions of type (10) we get:

(a) for equation (6)

$$\frac{dp_n^2}{dt}=\frac{DSc^2}{\lambda Q_v(\Delta x)^2}(p_{n-1}^2-2p_n^2+p_{n+1}^2) \tag{11}$$

(b) for equation (9)

$$\frac{\mathrm{d}Q^*_{v(x=0)}}{\mathrm{d}t} = -\frac{S}{2\rho^*\Delta x}(3p_{x=0} - 4p_1 + p_2) = \frac{\lambda\rho^* c^2}{2DS}\frac{Q^{*2}_{v(x=0)}}{p_{x=0}} \tag{12}$$

$$\frac{\mathrm{d}Q^*_{v,n}}{\mathrm{d}t} = -\frac{S}{2\rho^*\Delta x}(p_{n-1} - p_{n+1}) - \frac{\lambda\rho^* c^2}{2DS}\frac{Q^{*2}_{v,n}}{p_n} \tag{13}$$

$$\frac{\mathrm{d}p_n}{\mathrm{d}t} = \frac{\rho^* c^2}{2S\,\Delta x}(Q^*_{v,n-1} - Q^*_{v,n+1}) \tag{14}$$

$$\frac{\mathrm{d}p_N}{\mathrm{d}t} = \frac{\rho^* c^2}{2S\,\Delta x}(-Q^*_{v,N-2} + 4Q^*_{v,N-1} - 3Q^*_{v,N}) \tag{15}$$

where $n = 1, 2, \ldots, N$ (N is the number of discrete points chosen). For equation (6) it is necessary to determine value of pressure p_N using the boundary condition

$$Q^*_{v,N} = f(t) \tag{16}$$

For this reason the steady-state flow was assumed along the last pipeline discretization section, i.e. between x_{N-1} and x_N. Thus

$$Q^*_{v,N} = \sqrt{\left(\frac{p^2_{N-1} - p^2_N}{R_{N-1,N}}\right)} \tag{17}$$

where

$$R_{N-1,N} = \frac{\Delta x_{N-1,N} TsZ}{(389640 D^{8/3}_{N-1,N})^2}$$

p_{N-1}, p_N is expressed in MPa and $\Delta x_{N-1,N}$, $D_{N-1,N}$ in m.

Finally

$$p_N^{2(k+1)} = p_{N-1}^{2(k)} - R_{N-1,N} Q^{*2(k+1)}_{v,N} \tag{18}$$

where k is the number of time levels. The discrete formulae of equations (6) and (9) were solved by means of the Runge–Kutta fourth order method according to the following relation:

$$x(t+\Delta t) = x(t) + \tfrac{1}{6}K_1 + \tfrac{1}{3}(K_2 + K_3) + \tfrac{1}{6}K_4 \tag{19}$$

where

$$K_1 = f[x(t)]\,\Delta t$$
$$K_2 = f[x(t) + \tfrac{1}{2}K_1]\,\Delta t$$
$$K_3 = f[x(t) + \tfrac{1}{2}K_2]\,\Delta t$$
$$K_4 = f[x(t) + K_3]\,\Delta t$$

For equations (6) and (9) $x = p^2$ and $x = p$, respectively. The investigations have shown that the pipeline dynamics should be defined by means of a linear equation with respect to p^2 (equation (6)).

In Figure 2 there are shown changes of pressure $p(L, t)$ for two models of a pipeline with $L = 10^5$ m, $D = 0{\cdot}7$ m. It was assumed that $p(0, t) = 5{\cdot}066$ MPa = const and $Q^*_v(L, t)$ is as in

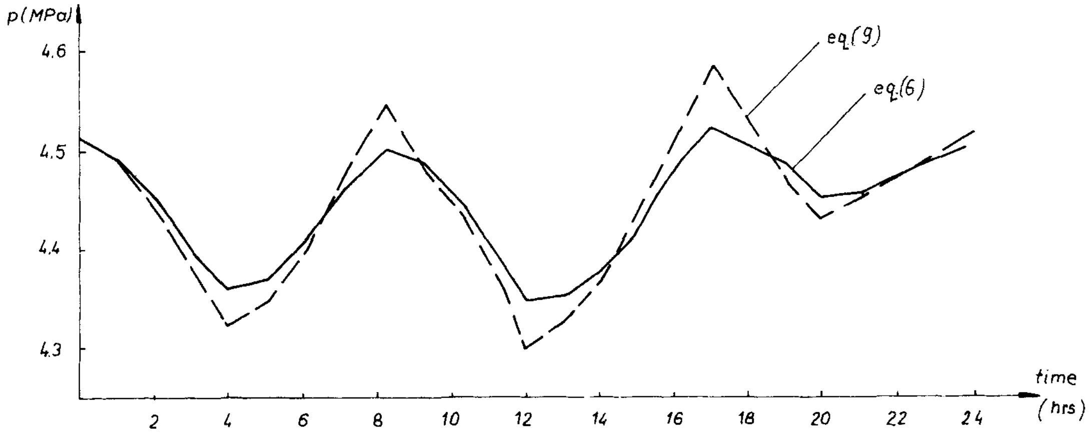

Figure 2. Changes of pressure for two models of a pipeline: $L = 10^5$ m, $D = 0{\cdot}7$ m

Figure 1. The following was obtained:

$$\sigma_0 = \max_t \left(\frac{|p_a - p_b|}{p_a}\right) 100 \text{ per cent} = 1{\cdot}2 \text{ per cent} \tag{20}$$

where p_a, p_b are the values of the pressure in discrete time computed using equations (6) and (9), respectively.

Investigations which were made for $L = 5 \times 10^4$ m, and $L = 10^4$ m ($D = 0{\cdot}7$ m) have shown that

$$\sigma_0 < 3{\cdot}0 \text{ per cent}$$

In each case the computation time was smaller by about 20 per cent for model (6) than for model (9). The comparison between the two above models was also made in conditions of very rapid changes of load; $Q_v^*(L, t)$; $p(L, t)$ for the pipeline was computed with $L = 10^4$ m, $D = 0{\cdot}6$ m and $Q_v^*(L, t)$ as in Figure 3. It was assumed that $p(0, t) = 4{\cdot}901$ MPa = const. Results are shown in Figure 4. Even in these conditions $\sigma_0 < 4$ per cent this thus confirms the correctness of our choosing model (6)—this equation being a compromise between explicitness and the calculation time that is necessary for its solution.

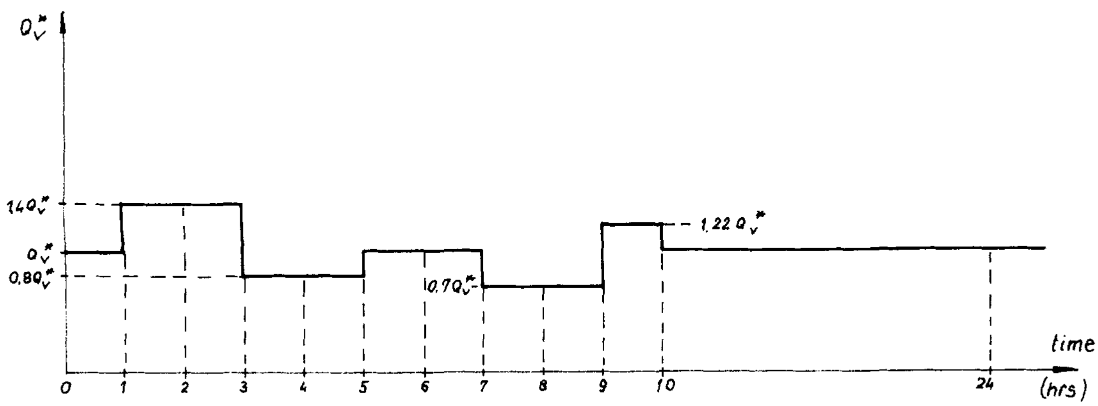

Figure 3. Changes of flow with time (boundary condition)

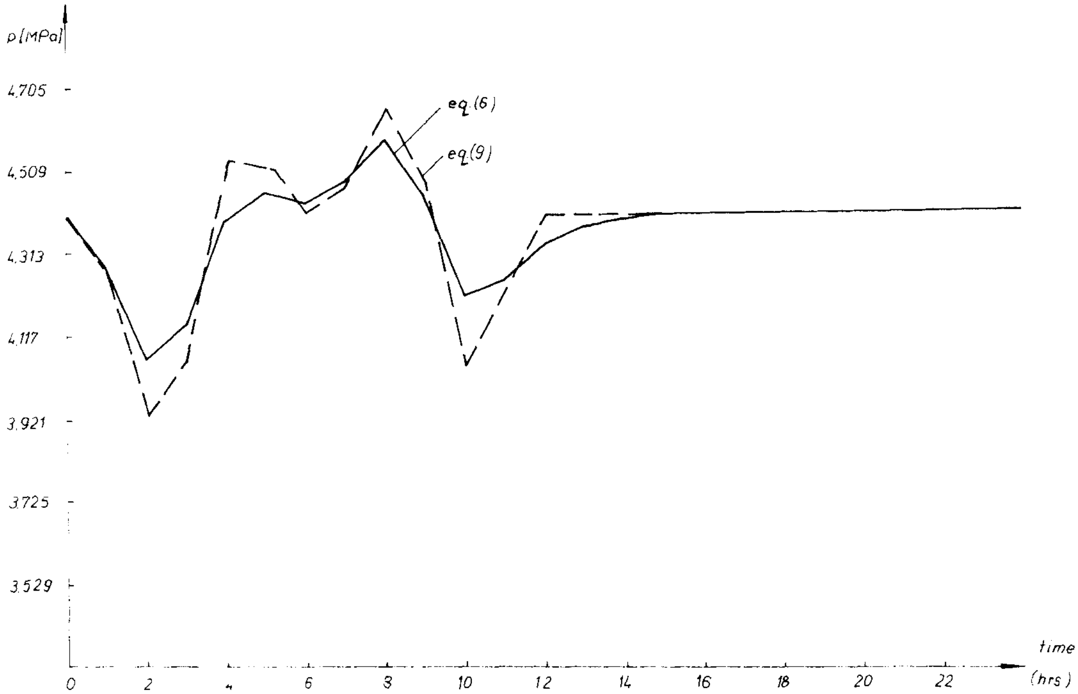

Figure 4. Changes of pressure for two models of a pipeline: $L = 10^4$ m, $D = 0{\cdot}6$ m

METHOD OF SOLUTION OF TRANSIENT FLOW EQUATION

Investigations[2] have shown that the best numerical scheme which meets the criteria of accuracy and relatively small computation time for equation (6) is the scheme shown in Figure 5. Using the above scheme we transform equation (6) into

$$\frac{P_n^{k+1}-P_n^k}{\Delta t}=a\frac{P_{n-1}^{k+1}-2P_n^{k+1}+P_{n+1}^{k+1}}{(\Delta x)^2}+O(\Delta t, (\Delta x)^2) \tag{21}$$

Equation (21) can be written in matrix form as

$$\mathbf{BP}^{k+1}=r\mathbf{R}^{k+1}+\mathbf{CP}^k \tag{22}$$

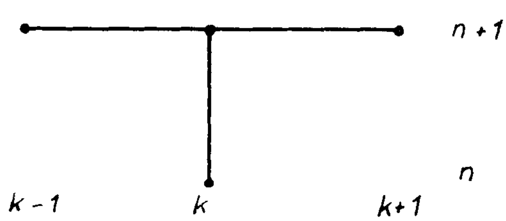

Figure 5. Implicit scheme, unrestricted stability

where

$$P_n^k = p^2(x_n, t_k)$$

$$\mathbf{P}^k = \begin{bmatrix} P_1^k \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_{N-1}^k \end{bmatrix} \qquad \boldsymbol{R}^{k+1} = \begin{bmatrix} P_0^{k+1} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ P_N^{k+1} \end{bmatrix}$$

$$\dim \mathbf{P} = \dim \mathbf{R} = (N-1) \times 1$$

N = number of discrete points chosen

$$r = \frac{a\,\Delta t}{(\Delta x)^2}$$

where Δt is the discrete time section and Δx the discrete section

$$\mathbf{B} = \begin{bmatrix} 1+2r, & -r, & 0, & \ldots\ldots\ldots\ldots & 0 \\ -r, & 1+2r, & -r, & 0, \ldots\ldots\ldots & 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & & \\ 0 & \ldots\ldots\ldots\ldots & 0, & -r, \quad 1+2r, & -r \\ 0 & \ldots\ldots\ldots\ldots\ldots\ldots & & 0, \quad -r, & 1+2r \end{bmatrix}$$

$\mathbf{C} = \mathbf{I}$ (unit matrix)

For an implicit scheme the unknown values of $\mathbf{P}$ at any time level are found by solving a set of algebraic equations:

$$\mathbf{A}\mathbf{P}^{k+1} = \mathbf{b} \tag{23}$$

Equations (22) take a tridiagonal form (elements occur only on the main diagonal and on one subdiagonal above and below). This system of equations was solved using the Thomas algorithm.[3]

METHOD OF COMPUTATION OF NODE PRESSURES

Graph theory has been used to represent the network structure. The graph nodes represent the joints between pipes, whereas the edges represent the pipelines. The values of the pressures at discrete points along the pipe from 1 to $N-1$ were computed using the numerical scheme (Figure 5). The values of node pressures were determined using the equation:

$$\frac{\mathrm{d}m_j}{\mathrm{d}t} = \sum_{i=1}^{n} B_{ij} Q_{\mathrm{m}(ij)} - Q_{\mathrm{m}(j)} \tag{24}$$

where

$Q_{\mathrm{m}(j)}$ is the demand at the jth node (mass flow)
m is the mass of gas in the node
n is the number of pipes connected to the node

$Q_{m(ij)}$ is the mass flow into or out of the jth node in the ith pipe connected to the jth node

$B_{ij}=1$ if the flow $Q_{m(ij)}$ is into the jth node

$B_{ij}=-1$ if the flow $Q_{m(ij)}$ is out of the jth node

Next:

$$\frac{dm}{dt}=\frac{dV\rho}{dt}=\frac{V}{c^2}\frac{dp}{dt} \tag{25}$$

where ρ is the gas density and V is the volume at the node. Using (25) we may reformulate equation (24) in the following form

$$\frac{V}{c^2}\frac{dp_j}{dt}=\sum_{i=1}^{n}B_{ij}Q_{m(ij)}-Q_{m(j)} \tag{26}$$

Seeing that

$$\frac{dp_j}{dt}\simeq\frac{p_j^{k+1}-p_j^k}{\Delta t}$$

we can express equation (26) as

$$p_j^{k+1}-p_j^k-\frac{\Delta tc^2}{V}\left(\sum_{i=1}^{n}B_{ij}Q_{m(ij)}^{k+1}-Q_{m(j)}^{k+1}\right)=f(p_j^{k+1}) \tag{27}$$

where

$$Q_{m(ij)}^{k+1}=Q_{v(ij)}^{*(k+1)}\rho^*$$

$$Q_{m(j)}^{k+1}=Q_{v(j)}^{*(k+1)}\rho^*$$

We assume that we have steady state flow along each element Δx_i adjacent to the jth node. So

$$Q_{v(ij)}^{*(k+1)}=389640D_{ij}^{8/3}\sqrt{\left(\frac{p_i^{2(k)}-p_j^{2(k+1)}}{TsZ\,\Delta x_i}\right)}$$

The equation

$$f(p_j^{k+1})=0 \tag{28}$$

was solved for p_j^{k+1} using the bisection method.[4] This is slower than the Newton–Raphson method, but gives correct results even if the initial value is far away from the correct solution.

THE RESULTS OF INVESTIGATIONS

The algorithm for simulating transient flow in a gas network calculates pressures at nodes and at discrete points along the pipes at each subsequent time $t+\Delta t$. The latter pressures are calculated using equation (21) and the former pressures are then calculated using the bisection method. The computational cycle is repeated $T/\Delta t$ times.

Simulation of the dynamic gas flow has been conducted for three networks with the following structures:

(i) 6 nodes, 8 edges
(ii) 22 nodes, 36 edges
(iii) 50 nodes, 78 edges.

$$D=0{\cdot}5\text{ m},\qquad L\in[100\text{ m};\ 14\times10^4\text{ m}]$$

It was assumed that the loads in each node are discrete periodic functions with a discrete interval $\Delta t = 2$ h, a period $T = 24$ h and linear in each interval. Nodal pressures and pressures at discrete points along each pipe were computed for given pressure values at the sources. The simulation period was 72 h. The initial investigation verified that this period was ample enough for the model to 'forget' the initial conditions. Gas networks were simulated for $\Delta t = 600$ s and $N = 4$ using a CDC 7600 computer. The results were

(a) for 6 nodes: 1·824 s
(b) for 22 nodes: 8·853 s
(c) for 50 nodes: 19·389 s.

SIMULATION OF TRANSIENT GAS FLOWS IN NETWORKS WITH COMPRESSORS

The basic control elements in a high pressure gas pipeline network are compressor stations. The jth compressor raises the suction pressure p_{sj} to a higher discharge pressure p_{dj}. This higher outlet pressure then provides a pressure gradient maintaining the flow in the next pipeline segment. Under the assumption of adiabatic compression, the horsepower required to maintain a specified compression ratio (p_{dj}/p_{sj}) for a specified flow Q_j is given by

$$(\mathrm{HP})_j = A_j Q_j \left[\left(\frac{p_{dj}}{p_{sj}} \right)^{R_j} - B_j \right] \tag{29}$$

where

p_{dj} is the discharge pressure for the jth compressor
p_{sj} is the suction pressure for the jth compressor
Q_j is the flow through the jth compressor
$(\mathrm{HP})_j$ is the horsepower required to achieve compression ratio (p_{dj}/p_{sj}) and flow Q_j at compressor j

and A_j, B_j, R_j are constants for the jth compressor.

The most important variables used in network simulation and associated with the compressor are the suction and discharge pressures and the flow through the compressor. Usually one of these quantities is being controlled to a given set value. In practice the outlet pressures or flow rates of the compressors are often controlled. It was assumed for simulation of transient gas flows in networks with compressors that

$$V = V_1 \cup V_2 \tag{30}$$

where V is the set of graph nodes, V_1 is the subset of main graph nodes, and V_2 is the subset of auxiliary graph nodes. The main graph nodes represent the joints of the pipes. Each compressor station is indicated by two nodes, the input node and the output node.

The set of graph edges is also divided:

$$E = E_1 \cup E_2 \tag{31}$$

where E_1 is the subset of pipes which supply compressor stations and E_2 is the subset of other pipes.

The algorithm for the simulation of transient gas flow in a network with compressor stations carries out the following actions for each interval Δt:

(a) computes values of nodes pressure for $v_i \in V_1$ using the bisection method

(b) computes values of pressure along each pipe for $e_i \in E_2$ assuming that $p_d = p_{set}$ and using the implicit numerical scheme characterized above,

(c) computes values of flow through each compressor station Q^*_{vT}, using the following equation

$$Q^*_{vT} = 389640 D^{8/3} \sqrt{\left(\frac{p_d^2 - p_1^{2(k+1)}}{TsZ\,\Delta x}\right)} \tag{32}$$

where k is the number of the time level. (This means that the flow through a compressor is evaluated using the discharge pressure and the pressure at the adjacent discretization point along the discharge pipe.)

(d) computes for $v_i \in V_2$ the values of suction pressure for the time level $(k+1)$ using the equation:

$$D_s^{8/3} \sqrt{\left(\frac{p_{N-1}^{2(k)} - p_s^{2(k+1)}}{\Delta x_s}\right)} = D_d^{8/3} \sqrt{\left(\frac{p_d^2 - p_1^{2(k+1)}}{\Delta x_t}\right)} \tag{33}$$

where D_s is the diameter of the suction pipe, D_d is the diameter of the discharge pipe, Δx_s is the interval of discretization of the suction pipe, and Δx_d is the interval of discretization of the discharge pipe

(e) computes values of pressure along the pipes for $e_i \in E_1$.

For simulation of a gas network with 21 nodes and 2 compressor stations, with a varying load in each node ($Q^*_v(L, t)$ for $t \in [0, 24\text{ h}]$) and assuming $N = 4$, $\Delta t = 600$ s, the operation period was 3·5 s (CDC-7600).

CONCLUSIONS

The present work has resulted in the following advantages:

(i) The model described above has been verified on the basis of measurement data obtained in a real gas transmission system. The experiments have shown that this model can be applied in a dispatching centre.

(ii) A computer program is very fast and the accuracy is satisfactory for most applications even under large transient pertubations.

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